In the first chapter, we learned about how things move from place to place and began to develop an understanding of energy, an important conserved quantity. But motion doesn’t always involve a change of position, and energy isn’t the only conserved quantity that exists in nature. In this chapter, we’ll take a look at a second type of motion—rotation—and at two other conserved quantities—momentum and angular momentum. Spinning objects are quite common, and we’ll find it useful to understand their laws of motion before proceeding much further. Once we get those additional concepts under our belts, we’ll be ready to explore the physics behind a broad assortment of mechanical objects.

**EXPERIMENT: A Spinning Pie Dish**

Spinning a dish on the top of a narrow post seems like a simple activity. But don’t let its uncomplicated appearance deceive you: there are lots of physics involved in keeping the dish turning, in gradually slowing it down, and in preventing it from falling off the post.

An easy way to experiment with a spinning dish is to tape a pencil vertically to the edge of a table or chair so that its eraser projects several inches upward into the air. To avoid wobbling problems, the tape should hold the pencil rigidly in place, and the table or chair should be sturdy and stable.

Now prepare to balance a metal pie dish on the eraser before giving it a twist. If you don’t have a pie dish, you can use a Frisbee, a deep plastic plate, or a shallow plastic bowl instead. Be creative. You probably have something that will work; just don’t use Grandma’s heirloom porcelain unless you’re willing to face the possible consequences.
The Laws of Motion

The first thing you’ll need to do is to balance the dish on the eraser. Try to predict whether this task will be easy or hard. Will it matter whether the dish opens up or down? Observe what happens when you set the dish on the eraser. Did you verify your predictions?

Find an arrangement in which the dish balances well and then give the dish a gentle spin. What sort of influence do you have to exert on the dish to start it rotating? If the dish doesn’t wobble and the pencil remains stationary, the dish should spin for a while before coming to a stop. Once you’re no longer touching the dish, what keeps it turning? On the other hand, why doesn’t it keep turning forever?

Now flip the pencil over so that its sharp point projects upward. What will happen when you place the dish on that point? Will the dish still balance? When you spin the dish, will it turn for a longer or shorter time than on the bare eraser? Try to measure how changing the pivot on which the dish spins affects the duration of that spin. If the dish is soft and the point digs into it, protect the dish’s bottom by taping a coin to it. How does this improved pivot affect the dish’s rotation? Can you prolong the spin by taping weights around the outer edge of the dish? Is there a way to get the dish spinning just by blowing on it? Can you relate this motion to that of a spinning skater or a bicycle wheel?

Chapter Itinerary

We’re going to explore the laws of rotational motion and two new conserved quantities in the context of three everyday objects: (1) seesaws, (2) wheels, and (3) bumper cars. In seesaws, we’ll look at twists and turns, and see how two children manage to rock a seesaw back and forth. In wheels, we’ll examine how friction affects motion and learn how wheels make a vehicle more mobile. In bumper cars, we’ll learn the physics behind collisions and uncover some of the simple rules that govern what initially appear to be complicated motions. For a more complete preview of what we’ll examine in this chapter, flip ahead to the Chapter Summary on p. 74.
The ramp that we examined in Section 1.3 is only one tool that provides mechanical advantage. In this section, we’ll look at another such device: the type of lever known as a seesaw. As we discuss seesaws, we’ll revisit many of the laws of motion that we encountered in the previous chapter. However, we’ll see these laws in a new context: rotational motion.

**Questions to Think About:** A playground seesaw only balances when the children riding it are properly situated. What do we mean by a balanced seesaw? Why does it matter just where the children sit on the seesaw? What are they doing to make the balanced seesaw rock back and forth? Who is doing work on whom as they rock?

**Experiments to Do:** To get a feel for how levers work, find a rigid ruler with a hole in its center—the kind that can be clipped into a three-ring binder. If you support the ruler by putting the tip of an upright pencil into the central hole, you’ll find that the ruler balances; that is, it either remains stationary, at whatever orientation you choose, or rotates steadily about the central hole. (Eventually, the ruler comes to rest because of friction, a detail that we’ll continue to ignore for now.) Now, push on one end of the ruler. What happens? Try pushing the ruler’s end toward its central hole. What happens then? What is the most effective way to make the ruler spin?

Now lay the pencil on a table and place the ruler flat on top of it so that the pencil and the ruler are at right angles, or perpendicular, to each other. If you center the ruler on the pencil, the ruler will balance. Load the two ends of the ruler with coins or other
small weights, trying as you do to keep the ruler balanced. Try placing the coins at different positions relative to the pencil. Is there any way you can balance a light weight on one end with a heavy weight on the other end?

The Seesaw

Any child who has played on a seesaw with friends of different sizes knows that the toy works best for two children of roughly the same weight (Fig. 2.1.1a). Evenly matched riders balance each other, and this balance allows them to rock back and forth easily. In contrast, when a light child tries to play seesaw with a heavy child, the heavy child’s side of the seesaw drops rapidly and hits the ground with a thud (Fig. 2.1.1b). The light child is tossed into the air.

There are several solutions to the heavy child/light child problem. Of course, two light children could try to balance one heavy child. But most children eventually figure out that if the heavy child sits closer to the seesaw’s pivot, the seesaw will balance (Fig. 2.1.1c). The children can then make the seesaw tip back and forth easily, just as it does when two evenly matched children ride at its ends. This is a pretty useful trick, and we’ll explore it later in this section. First, though, we’ll need to look carefully at the nature of rotational motion.

For simplicity, let’s ignore the mass and weight of the seesaw itself. There are then only three forces acting on the seesaw shown in Fig. 2.1.1: two downward forces (the weights of the two children) and one upward force (the support force of the central pivot). Seeing those three forces, we may immediately think about net forces and begin to look for some overall acceleration of this toy and its riders. But we know that the seesaw remains where it is in the playground and isn’t likely to head off for Kalamazoo or the center of the earth anytime soon. Because the seesaw’s fixed pivot always provides just enough upward or sideways force to keep the seesaw from accelerating as a whole, the seesaw always experiences zero net force and never leaves the playground. Overall movement of an object from one place to another is called translational motion. While the seesaw never experiences this kind of motion, it can turn around the pivot, and thus it experiences a different kind of motion. Motion around a fixed point (which prevents translation) is called rotational motion. The hands of a clock experience rotational motion as they go around in a circle.

Rotational motion is what makes a seesaw interesting. The whole point of a seesaw is that it can rotate so that one child rises and the other descends. (You may not think of going up and down as rotating, but if the ground weren’t there, the seesaw would be able to rotate in a big circle.) What causes the seesaw to rotate, and what observations can we make about the process of rotation?

To answer those questions, we’ll need to examine several new physical quantities associated with rotation and explore the laws of rotational motion that relate them to one another. We’ll do these things both by studying the workings of seesaws and other rotating objects and by looking for analogies between translational motion and rotational motion.

Imagine holding onto the seesaw in Fig. 2.1.1a to keep it level for a moment while the child on the left climbs off the seesaw. Now imagine letting go of the seesaw. As soon as you let go, the seesaw will begin to rotate, and the child on the right will descend toward the ground. The seesaw’s motion will be fairly slow at first, but it will move more and more quickly until that child strikes the ground with a teeth-rattling thump.

If we focus only on the rotation itself, we might describe the motion of the seesaw in the following way:
“The seesaw starts out not rotating at all. When we release the seesaw, it begins to rotate clockwise. The seesaw’s rate of rotation increases steadily in the clockwise direction until the moment the seesaw strikes the ground.”

This description sounds a lot like the description of a falling ball released from rest:

“The ball starts out not moving at all. When we release the ball, it begins to move downward. The ball’s rate of translation increases steadily in the downward direction until the moment the ball strikes the ground.”

The statement about the seesaw involves rotational motion, while the statement about the ball involves translational motion. Their similarity isn’t a coincidence; the concepts and laws of rotational motion have many analogies in the concepts and laws of translational motion. The familiarity that we’ve acquired with translational motion will help us examine rotational motion.

Check Your Understanding #1: Wheel of Fortune Cookies

The guests at a large table in a Chinese restaurant use a revolving tray, a lazy Susan, to share the food dishes. How does the lazy Susan’s motion differ from that of the passing dessert cart?

The Motion of a Dangling Seesaw

In the previous chapter we looked at the concept of translational inertia, which holds that a body in motion tends to stay in motion and a body at rest tends to stay at rest. This concept led us to Newton’s first law of translational motion. Inserting the word “translational” here is a useful revision because we’re about to encounter analogous concepts associated with rotational motion. We’ll begin that encounter by observing a seesaw that’s free of outside rotational influences. We’ll then examine how the seesaw responds to outside influences such as its pivot or a handful of young riders. Because of the similarities between rotational and translational motions, this section will closely parallel our earlier examination of skating and falling balls.

Let’s suppose that your local playground is installing a new seesaw and that this seesaw is presently dangling from a rope. The rope is attached to the middle of the seesaw in such a way that it supports the seesaw’s weight but exerts no other influences on the seesaw. Most importantly, let’s suppose that the dangling seesaw can spin and pivot with complete freedom—nothing pushes on it or twists it—and that the rope doesn’t get tangled or in the way (Fig. 2.1.2). This dangling seesaw is free to turn in any direction, even completely upside down. You, the observer, are standing motionless near the seesaw. When you look over at the seesaw, what does it do?

If the seesaw is stationary, then it will remain stationary. However, if it’s rotating, it will continue rotating at a steady pace, about a fixed line in space. What keeps the seesaw rotating? Its rotational inertia. A body that’s rotating tends to remain rotating; a body that’s not rotating tends to remain not rotating. That’s how our universe works.

To describe the seesaw’s rotational inertia and rotational motion more accurately, we’ll need to identify several physical quantities associated with rotational motion. The first is the seesaw’s orientation. At any particular moment, the seesaw is oriented in a certain way—that is, it has an angular position. Angular position describes the seesaw’s
Seesaws

orientation relative to some reference orientation; it can be specified by determining how far the seesaw has rotated away from its reference orientation and the axis or line about which that rotation has occurred. The seesaw’s angular position is a vector quantity of relatively minor importance, pointing along the rotation axis with a magnitude equal to the rotation angle (Fig. 2.1.3).

The SI unit of angular position is the **radian**, the natural unit for angles. It’s a natural unit because it follows directly from geometry, not from an arbitrary human choice or convention the way most units do. Geometry tells us that a circle of radius 1 has a circumference of $2\pi$. By letting arc lengths around that circle’s circumference specify angles, we are using radians. For example, there are $2\pi$ radians (or 360°) in a full circle and $\pi/2$ radians (or 90°) in a right angle. Since the radian is a natural unit, it is often omitted from calculations and derived units.

If the seesaw is rotating, then its angular position is changing; in other words, it has an angular velocity. **Angular velocity** is our first important vector quantity of rotational motion and measures how quickly the seesaw’s angular position changes; it consists of the angular speed at which the seesaw is rotating and the axis about which that rotation proceeds. The seesaw’s **angular speed** is its change in angle divided by the time required for that change:

$$\text{angular speed} = \frac{\text{change in angle}}{\text{time}}.$$ 

The SI unit of angular velocity is the **radian-per-second** (abbreviated $1/s$).

The seesaw’s **axis of rotation** is the line in space about which the seesaw is rotating. But just knowing that line isn’t quite enough: is the seesaw rotating clockwise or counterclockwise?

To resolve this ambiguity, we take advantage of the fact that any line has two directions to it. Once we have identified the line about which the seesaw is rotating, we can look down that line at the seesaw from either direction. From one direction, the seesaw appears to be rotating clockwise; from the other direction, counterclockwise. By convention, we choose the direction in which the seesaw appears to be rotating clockwise and say that the seesaw’s rotation axis points away from our eye toward the seesaw. This convention is called the **right-hand rule** because if the fingers of your right hand are curling around the axis in the way the seesaw is rotating, then your thumb is pointing along the seesaw’s rotation axis (Fig. 2.1.4).

Remembering this convention isn’t as important as understanding why we must specify the direction about which rotation occurs when describing a rotating object’s angular velocity. Just as translational velocity consists of a translational speed and a direction in which the translational motion occurs, so angular velocity consists of a rotational speed and a direction about which the rotational motion occurs.

We’re now prepared to describe the rotational motion of the dangling seesaw. Because of its freedom from outside influences and its rotational inertia, its angular velocity is constant. The dangling seesaw just keeps on turning and turning, always at the same angular speed, always about the same axis of rotation.

As you might suspect, this observation isn’t unique to seesaws. It is **Newton’s first law of rotational motion**, which states that a rigid object that is not wobbling and is not subject to any outside influences rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation. The outside influences referred to in this law are called **torques**—a technical term for twists and spins. When you twist off the lid of a jar or spin a top with your fingers, you’re exerting a torque.

This law excludes objects that wobble or can change shape as they rotate because those objects have more complicated motions. They are covered instead by a more general principle—the conservation of angular momentum—that we’ll learn later on.
Newton’s First Law of Rotational Motion
A rigid object that is not wobbling and is not subject to any outside torques rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation.

Check Your Understanding #2: Going for a Spin
A rubber basketball floats in a swimming pool. It experiences zero torque, no matter which end of it is up. If you spin the basketball and then let go, how will it move?

The Seesaw’s Center of Mass
Even without visiting the playground, you can find many objects that are nearly free from torques: a baton thrown overhead by a baton twirler, for example, or a juggler’s club whirling through the air between two clowns. These motions, however, are complicated because those freely moving objects rotate and translate at the same time. The spinning baton travels up and down, the turning club arcs through the air, and, if the rope breaks, our seesaw will fall as it spins. How can we distinguish their translational motions from their rotational motions?

Once again, we can make use of a wonderful simplification of physics. There’s a special point in or near a free object about which all its mass is evenly balanced and about which it naturally spins—its center of mass. The axis of rotation passes right through this point so that, as the free object rotates, the center of mass doesn’t move unless the object has an overall translational velocity. The center of mass of a typical ball is at its geometrical center, while the center of mass of a less symmetrical object depends on how the mass of that object is distributed. You can begin to find a small object’s center of mass by spinning it on a smooth table and looking for the fixed point about which it spins (Fig. 2.1.5).

Center of mass allows us to separate an object’s translational motion from its rotational motion. As a juggler’s club arcs through space, its center of mass follows the simple path we discussed in Section 1.2 on falling balls (Fig. 2.1.6). At the same time, the club’s rotational motion about its center of mass is that of an object that’s free of outside torques: if it’s not wobbling, it rotates with a constant angular velocity.

In the course of this book we’ll encounter many objects that translate and rotate simultaneously, and it’s worth remembering that we can often separate these two motions by paying attention to an object’s center of mass. For example, the workers installing our seesaw will locate its pivot strategically at or very near the seesaw’s center of mass. As a result, the pivot will prevent any translational motion of the seesaw while permitting nearly free rotational motion of the seesaw about its center of mass, at least about one axis.

Check Your Understanding #3: Tracking the High Dive
When a diver does a rigid, open somersault off a high diving board, his motion appears quite complicated. Can this motion be described simply? How?
How the Seesaw Responds to Torques

The workers are eating lunch, so the seesaw is still hanging from the rope. Why can’t this dangling seesaw change its rotational speed or axis of rotation? Because it has rotational mass. Rotational mass is the measure of an object’s rotational inertia, its resistance to changes in its angular velocity. An object’s rotational mass depends both on its ordinary mass and on how that mass is distributed within the object. The SI unit of rotational mass is the kilogram-meter$^2$ (abbreviated kg·m$^2$). Because the seesaw has rotational mass, its angular velocity will change only if something twists it or spins it. In other words, it must experience a torque.

Torque—our second important vector quantity of rotational motion—has both a magnitude and a direction. The more torque you exert on the seesaw, the more rapidly its angular velocity changes. Depending on the direction of the torque, you can make the seesaw turn more rapidly or less rapidly or even rotate about a different axis. But how do you determine the direction of a particular torque? One way is to imagine exerting this torque on a stationary ball floating in water (Fig. 2.1.7a,b). The ball will begin to rotate, acquiring a nonzero angular velocity (Fig. 2.1.7c). The direction of this angular velocity is that of the torque. The SI unit of torque is the newton-meter (abbreviated N·m).

The larger an object’s rotational mass, the more slowly its angular velocity changes in response to a specific torque (Fig. 2.1.8). You can easily spin a basketball with the tips of your fingers because it has a smaller rotational mass than a bowling ball.

For clarity and simplicity, this book refers to the measure of an object’s rotational inertia as “rotational mass.” However, this quantity is known more formally as “moment of inertia.”
your fingers, but it’s much harder to spin a bowling ball. The bowling ball’s larger rotational mass comes about primarily because it has a greater ordinary mass than the basketball.

But rotational mass also depends on an object’s shape, particularly on how far each portion of its ordinary mass is from the axis of rotation. The farther a portion of mass is from that axis, the more rapidly it must accelerate as the entire object undergoes angular acceleration and the more leverage it has with which to oppose that acceleration. We’ll examine levers shortly, but the consequence of these two effects of distance from the rotation axis is that each portion of mass contributes to the object’s rotational mass in proportion to the square of its distance from that axis. That’s why an object that has most of its mass located near the axis of rotation will have a much smaller rotational mass than an object of the same mass that has most of its mass located far from that axis. Thus a ball of pizza dough has a smaller rotational mass than the finished pizza. And the bigger the pizza gets, the harder it is to start or stop spinning.

Because an object’s rotational mass depends on how far its mass is from the axis of rotation, changes in the axis of rotation are likely to change its rotational mass. For example, less torque is required to spin a tennis racket about its handle (Fig. 2.1.9a) than to flip the racket head-over-handle (Fig. 2.1.9b). When you spin the tennis racket about its handle, the axis of rotation runs right through the handle so that most of the racket’s mass is fairly close to the axis and the rotational mass is small. When you flip the tennis racket head-over-handle, the axis of rotation runs across the handle so that both the head and the handle are far away from the axis and the rotational mass is large. The tennis racket’s rotational mass becomes even larger when you hold it in your hand and make it rotate about your shoulder rather than its center of mass (Fig. 2.1.9c).

When something exerts a torque on the dangling seesaw, its angular velocity changes; in other words, it undergoes angular acceleration, our third important vector quantity of rotational motion. Angular acceleration measures how quickly the seesaw’s angular velocity changes. It’s analogous to acceleration, which measures how quickly an object’s translational velocity changes. Just as with acceleration, angular acceleration involves both a magnitude and a direction. An object undergoes angular acceleration when its angular speed increases or decreases or when its angular velocity changes directions. The SI unit of angular acceleration is the radian-per-second² (abbreviated 1/s²).

There is a simple relationship between the torque exerted on the seesaw, its rotational mass, and its angular acceleration. The seesaw’s angular acceleration is equal to the torque exerted on it divided by its rotational mass or, as a word equation,

\[
\text{angular acceleration} = \frac{\text{torque}}{\text{rotational mass}}. \tag{2.1.1}
\]

The seesaw’s angular acceleration, as we’ve seen, is in the same direction as the torque exerted on it.

This relationship is Newton’s second law of rotational motion. Structuring the relationship this way distinguishes the causes (torque and rotational mass) from their effect (angular acceleration). Nonetheless, it has become customary to rearrange the relationship to eliminate the division. In its traditional form, the relationship can be written in a word equation:

\[
\text{torque} = \text{rotational mass} \cdot \text{angular acceleration}, \tag{2.1.2}
\]

in symbols:

\[
\tau = I \cdot \alpha,
\]
Seesaws

and in everyday language:

*Spinning a marble is much easier than spinning a merry-go-round.*

It resembles Newton’s second law of translational motion (force = mass \* acceleration), except that torque has replaced force, rotational mass has replaced mass, and angular acceleration has replaced acceleration. This new law doesn’t apply to wobbling objects, however, because they’re being affected by more than one rotational mass simultaneously (see the discussion of tennis rackets above) and follow a much more complicated law.

### Newton’s Second Law of Rotational Motion

The torque exerted on an object that is not wobbling is equal to the product of that object’s rotational mass times its angular acceleration. The angular acceleration points in the same direction as the torque.

Because it’s an equation, the two sides of Eq. 2.1.1 are equal. Any change in the torque you exert on the seesaw must be accompanied by a proportional change in its angular acceleration. As a result, the harder you twist or spin the seesaw, the more rapidly its angular velocity changes.

We can also compare the effects of a specific torque on two different rotational masses. Equation 2.1.1 indicates that a decrease in rotational mass must be accompanied by a corresponding increase in angular acceleration. If we replace the playground seesaw with one from a dollhouse, the rotational mass will decrease and the angular acceleration will increase. The angular velocity of a doll’s seesaw thus changes more rapidly than the angular velocity of a playground seesaw when the two experience identical torques.

In summary:

1. Your angular position indicates exactly how you’re oriented.
2. Your angular velocity measures how quickly your angular position changes.
3. Your angular acceleration measures how quickly your angular velocity changes.
4. In order for you to undergo angular acceleration, something must exert a torque on you.
5. The more rotational mass you have, the less angular acceleration you experience for a given torque.

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<th>SI Unit</th>
<th>English Unit</th>
<th>SI → English</th>
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<tr>
<td>Torque</td>
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<td>1 ft·lbf = 1.3558 N·m</td>
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<tr>
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<td>pound-foot^2 (lbm·ft^2)</td>
<td>1 kg·m^2 = 23.730 lbm·ft^2</td>
<td>1 lbm·ft^2 = 0.042140 kg·m^2</td>
</tr>
</tbody>
</table>
This summary of the physical quantities of rotational motion is analogous to the one for translational motion on p. 43. Take a moment to compare the two.

Check Your Understanding #4: The Merry-Go-Round

The merry-go-round is a popular playground toy (see Fig. 2.1.8). Already challenging to spin empty, a merry-go-round is even harder to start or stop when there are lots of children on it. Why is it so difficult to change a full merry-go-round’s angular velocity?

Check Your Figures #1: Hard to Turn

Automobile tires are normally hollow and filled with air. If they were made of solid rubber, their rotational masses would be about 10 times as large. With the wheel lifted off the ground, how much more torque would an automobile have to exert on a solid tire to make it undergo the same angular acceleration as a hollow tire?

Forces, Torques, and Seesaws

The workers have finally installed the seesaw. They have mounted it on a pivot that passes directly through its center of mass, so that the pivot coincides with a natural rotation axis of the seesaw. The pivot thus supports the seesaw’s weight while leaving it free to obey Newton’s first law of rotational motion. That is, the unoccupied seesaw rotates with constant angular velocity about its pivot.

The unoccupied seesaw is balanced, meaning that it has zero torque on it. As a result, it experiences no angular acceleration. You might think that a balanced seesaw always remains horizontal, but that isn’t necessarily so. What is certain is that its angular velocity is constant. If it’s rotating, then it continues to rotate steadily about the pivot; if it’s stationary, then it remains stationary at its current tilt, whether horizontal or not.

To change the seesaw’s angular velocity, you must exert a torque on it. But how do you actually exert a torque? You put your hand on one end of the seesaw and push that end down (Fig. 2.1.10a). The seesaw begins to rotate, and your end soon hits the ground. You have exerted a torque on the seesaw.

But you started by exerting a force on the seesaw—you pushed on it—so forces and torques must be related somehow. Sure enough, a force can produce a torque and a torque can produce a force. To help us explore that relationship, let’s think of all the ways not to exert a torque on a seesaw.

What happens if you push on the seesaw right where the pivot passes through it (Fig. 2.1.10b)? Nothing—no angular acceleration. If you move a little away from the pivot, you can get the seesaw rotating but you have to push hard. You do much better to push on the end of the seesaw, where even a small force can start the seesaw rotating.

The distance from the pivot to the place where you push on the seesaw is called the lever arm; in general, the longer the lever arm, the less force it takes to cause a particular angular acceleration. Our first observation about producing a torque with a force is this: you obtain more torque by exerting that force farther from the pivot or axis of rotation. In other words, the torque is proportional to the lever arm.
Another ineffective way to start the seesaw rotating is to push its end directly toward or away from the pivot (Fig. 2.1.10c). A force exerted toward or away from the axis of rotation doesn’t produce any torque about that axis. At least a component of the force you exert must be perpendicular to the lever arm, which is actually a vector pointing along the seesaw’s surface from the pivot to the place where you push on the seesaw. Our second observation about producing a torque with a force is that you must exert at least a component of that force perpendicular to the lever arm. Only that component of force contributes to the torque. To produce the most torque, push perpendicular to the lever arm.

We can summarize these two observations as follows: the torque produced by a force is equal to the product of the lever arm times that force, where we include only the component of the force that is perpendicular to the lever arm. This relationship can be written as a word equation:

$$\text{torque} = \text{lever arm} \cdot \text{force perpendicular to lever arm}, \tag{2.1.3}$$

in symbols:

$$\tau = r \cdot F_L,$$

and in everyday language:

*When twisting an unyielding object, it helps to use a long stick.*

The directions of the force and lever arm also determine the direction of the torque. The three directions follow another right-hand rule (Fig. 2.1.11). If you point your right index finger in the direction of the lever arm and your bent middle figure in the direction of the force, then your thumb will point in the direction of the torque. Thus in Fig. 2.1.11a, the lever arm points to the right, the force points downward, and the resulting torque points into the page so that the seesaw undergoes angular acceleration in the clockwise direction. In Fig. 2.1.11b, the lever arm has reversed directions and so has the torque.

What happens if you and a friend push down simultaneously on both seats at once? Then you produce two torques on the seesaw about its pivot, and these torques have opposite directions. The seesaw responds to the net torque it experiences, the sum of all the individual torques on the seesaw. Since your two torques oppose one another, they at least partially cancel. If you carefully exert identical downward forces at identical distances from the pivot, the magnitudes of the two torques will be exactly equal and the torques will sum to zero. The seesaw will experience zero net torque, and it will be balanced.

This observation explains the need for careful seating of the children on the seesaw. Each child’s weight exerts a downward force on the seesaw and by properly distributing those weights on both sides of the pivot, the torques that they produce can be made to sum to zero. With zero net torque about its pivot, the seesaw balances.

In fact, the weight of the seesaw itself is balanced in this manner. While each end’s weight exerts a torque on the seesaw, those two torques sum to zero and have no overall effect on the seesaw’s rotation.

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**Check Your Understanding #5: Cutting Up Cardboard**

When you cut cardboard with a pair of scissors, it’s best to move the cardboard as close as possible to the scissors’ pivot. Explain.
You're trying to remove some rusty screws from your refrigerator, using an adjustable wrench with a 0.2-meter (20-centimeter) handle. Although you push as hard as you can on the handle, you can't produce enough torque to loosen one of the screws. You have a 1.0-meter (100-cm) pipe that you can slip over the handle of the wrench to make the wrench effectively 1 meter long. How much more torque will you then be able to exert on the screw?

Net Torque and Mechanical Advantage

The amount of torque that a child’s weight produces on a seesaw depends on that child’s distance from the pivot. If the child sits on the pivot, the lever arm is zero and she produces no torque; but if she sits at the extreme end of the seesaw, the lever arm is long and she produces a large torque. She can adjust her torque by moving along the seesaw because the seesaw provides her with mechanical advantage. As we saw in Section 1.3, mechanical advantage appears when a device redistributes the amounts of force and distance used to produce a particular amount of work. The seesaw allows a small force exerted at its end to do the same work as a large force exerted near its pivot.

To see how mechanical advantage appears in a seesaw, think of what happens when two children sit on its ends. If two 5 year olds, each weighing 200 N (45 lbf), sit at opposite ends of the seesaw, 2 m (6.6 ft) from the pivot (Fig. 2.1.12a), each one exerts a torque of $400 \text{ N} \cdot \text{m}$ (300 ft-lbf) on the seesaw about its pivot (200 N \cdot 2 \text{ m} = 400 \text{ N} \cdot \text{m}). But because these torques are oppositely directed, they add to zero. The net torque on the seesaw is zero and the seesaw balances.

If you replace one of the 5 year olds with a 400-N (90-lbf) teenager, the teenager must sit at half the distance from the pivot (Fig. 2.1.12b). Doubling the force while halving the lever arm leaves the torque unchanged at $400 \text{ N} \cdot \text{m}$. The two children again produce equal but oppositely directed torques about the pivot, so that the net torque on the seesaw is zero and the seesaw balances. This effect explains how a small child at the end of the seesaw can balance a large child nearer the pivot.

With the seesaw balanced, nothing is accelerating. Each child experiences zero net force; the seesaw pushes up on the small child with a force of 200 N (45 lbf) and on the large child with a force of 400 N (90 lbf). Ultimately, it’s the small child’s 200-N (45-lbf) weight that gives rise to the 400-N (90-lbf) supporting force experienced by the large child. The seesaw’s mechanical advantage allows the small child to support and lift the much heavier child. This effect, where a small force on one part of a rotating system produces a large force elsewhere in that system, is an example of the mechanical advantage associated with levers.

Check Your Figures #2: A Few Loose Screws

You’re trying to remove some rusty screws from your refrigerator, using an adjustable wrench with a 0.2-meter (20-centimeter) handle. Although you push as hard as you can on the handle, you can’t produce enough torque to loosen one of the screws. You have a 1.0-meter (100-cm) pipe that you can slip over the handle of the wrench to make the wrench effectively 1 meter long. How much more torque will you then be able to exert on the screw?

Check Your Understanding #6: Pulling Nails

Some hammers have a special claw designed to remove nails from wood. When you slide the claw under the nail’s head and rotate the hammer by pulling on its handle, the claw pulls the nail out of the wood. The hammer’s head contacts the wood to form a pivot that’s about 10 times closer to the nail than to the handle. The torque you
exert on the hammer twists it in one direction, while the torque that the nail exerts on the hammer twists it in the opposite direction. The hammer isn’t undergoing any significant angular acceleration, so the torques must nearly balance. If you’re exerting a force of 100 N (22 lbf) on the hammer’s handle, how much force is the nail exerting on the hammer’s claw?

**Riding a Seesaw**

Each seesaw in Fig. 2.1.12 is balanced, meaning that the net torque on it is zero. Although each child’s weight exerts a torque on the seesaw, the two torques sum to zero. Since the seesaw experiences zero net torque and no angular acceleration, it continues rotating at constant angular velocity.

However, as it presently stands, a balanced seesaw should either remain motionless forever or else rotate endlessly in the same direction. Children are unlikely to wait motionless forever, and endless rotation implies that the children will be upside-down periodically. We’ve obviously neglected a few details.

What do the children do when the seesaw is motionless? To start the seesaw moving, they have to unbalance the seesaw. One of the children must change the torque she exerts on it. She can either change the downward force she exerts on the seesaw or change the distance between that force and the pivot. Actually, children change both the force and the lever arm frequently without even thinking about it. If a child leans inward, toward the pivot, the lever arm decreases and the child exerts less torque on the seesaw; as a result, the seesaw begins to rotate and the child rises. If the child pushes on the ground with his feet, the ground exerts an upward force on him, reducing the force and torque he exerts on the seesaw; again, the seesaw begins to rotate and the child rises.

So either by leaning or by pushing on the ground, the children can start an initially motionless, balanced seesaw rotating. Similarly, when one end of the seesaw hits the ground, the ground exerts a strong, upward support force on it. Located far from the pivot and almost perpendicular to that lever arm, this force produces a huge torque on the seesaw and abruptly stops it from rotating. The angular acceleration is so uncomfortably large that most children push on the ground with their feet to cushion the impact. The child on the ground can continue to push down with her feet until the seesaw rotates in the opposite direction. That child begins to rise and the other child descends. When the other end of the seesaw reaches the ground, this cycle begins again.

As they play on a seesaw, the two children frequently change the torques they exert on it so that it tips back and forth. During the moments when a child is pushing on the ground or leaning inward or outward to get a stationary seesaw moving, the seesaw is no longer balanced. A balanced seesaw has zero angular acceleration; it’s only by unbalancing the seesaw that the children can change the angular velocity of the seesaw.

**Check Your Understanding #7: Rocking the Boat**

Loading a large container ship requires some care in balancing the cargo and fastening it down firmly. The effective pivot about which the ship can rotate in the water is located roughly along the centerline of the ship, from its bow to its stern. Why is improperly fastened-down cargo so dangerous on such a ship, possibly causing it to capsize during a storm?
Like ramps and levers, wheels are simple tools that make our lives easier. But a wheel’s main purpose isn’t mechanical advantage, it’s overcoming friction. Up until now, we’ve ignored friction, looking at the laws of motion as they apply only in idealized situations. But our real world does have friction, and an object in motion tends to slow down and stop because of it. One of our first tasks in this section will therefore be to understand friction—though, for the time being, we’ll continue to neglect air resistance.

**Questions to Think About:** If objects in motion tend to stay in motion, why is it so hard to drag a heavy box across the floor? If objects should accelerate downhill on a ramp, why won’t a plate slide off a slightly tilted table? What makes the wheels of a cart turn as you pull the cart forward? How does spinning its wheels propel a car forward?

**Experiments to Do:** To observe the importance of wheels in eliminating friction, try sliding a book along a flat table. Give the book a push and see how quickly it slows down and stops. Which way is friction pushing on the book? Does the force that friction exerts on the book depend on how fast the book is moving? Let the book come to a stop. Is friction still pushing on the book when it isn’t moving? If you push gently on the stationary book, what force does friction exert on it?

Lay down three or four round pencils, parallel to one another and a few inches apart. Rest the book on top of the pencils and give the book a push in the direction that the pencils can roll. Describe how the book now moves. What do you think has caused the difference?

**Moving a File Cabinet: Friction**

When we imagined moving your friend’s piano into a new apartment back in Section 1.3, we neglected a familiar force—friction. Luckily for us, your friend’s piano had wheels on its legs, and wheels facilitate motion by reducing the effects of friction. We’ll focus
on wheels in this section. But first, to help us understand the relationship between wheels and friction, we’ll look at another item that needs to be moved—your friend’s file cabinet.

The file cabinet is resting on a smooth and level hardwood floor; it’s full of sheet music and weighs about 1000 N (225 lbf). Despite its large mass, you know that it should accelerate in response to a horizontal force, so you give it a gentle push toward the door. Nothing happens. Something else must be pushing on the file cabinet in just the right way to cancel your force and keep it from accelerating. Undaunted, you push harder and harder until finally, with a tremendous shove, you manage to get the file cabinet sliding across the floor. But the cabinet moves slowly even though you continue to push on it. Something else is pushing on the file cabinet, trying to stop it from moving.

That something else is friction, a force that opposes the relative motion of two surfaces in contact with one another. Two surfaces that are in relative motion are traveling with different velocities so that a person standing still on one surface would observe the other surface as moving. In opposing relative motion, friction exerts forces on both surfaces in directions that tend to bring them to a single velocity.

For example, when the file cabinet slides by itself toward the left, the floor exerts a rightward frictional force on it (Fig. 2.2.1). The frictional force exerted on the file cabinet, toward the right, is in the direction opposite the file cabinet’s velocity, toward the left. Since the file cabinet’s acceleration is in the direction opposite its velocity, the file cabinet slows down and eventually comes to a stop.

According to Newton’s third law of motion, an equal but oppositely directed force must be exerted by the file cabinet on the floor. Sure enough, the file cabinet does exert a leftward frictional force on the floor. However, the floor is rigidly attached to the earth, so it accelerates very little. The file cabinet does almost all the accelerating, and soon the two objects are traveling at the same velocity.

Frictional forces always oppose relative motion, but they vary in strength according to (1) how tightly the two surfaces are pressed against one another, (2) how slippery the surfaces are, and (3) whether or not the surfaces are actually moving relative to one another. First, the harder you press two surfaces together, the larger the frictional forces they experience. For example, an empty file cabinet slides more easily than a full one. Second, roughening the surfaces generally increases friction, while smoothing or lubricating them generally reduces it. Riding a toboggan down the driveway is much more interesting when the driveway is covered with snow or ice than when the driveway is bare asphalt. We’ll examine the third issue later on.

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**Check Your Understanding #1: The One That Got Away**

Your table at the restaurant isn’t level, and your water glass begins to slide slowly downhill toward the edge. Which way is friction exerting a force on it?

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**A Microscopic View of Friction**

As the file cabinet slides by itself across the floor, it experiences a horizontal frictional force that gradually brings it to a stop. But from where does this frictional force come? The obvious forces on the file cabinet are both vertical, not horizontal: the cabinet’s weight is downward and the support force from the floor is upward. How can the floor exert a horizontal force on the file cabinet?

The answer lies in the fact that neither the bottom of the file cabinet nor the top of the floor is perfectly smooth. They both have microscopic hills and valleys of various sizes.
The file cabinet is actually supported by thousands of tiny contact points, where the file cabinet directly touches the floor (Fig. 2.2.2). As the file cabinet slides, the microscopic projections on the bottom of the file cabinet pass through similar projections on the top of the floor. Each time two projections collide, they experience horizontal forces. These tiny forces oppose the relative motion and give rise to the overall frictional forces experienced by the file cabinet and floor. Because even an apparently smooth surface still has some microscopic surface structure, all surfaces experience friction as they rub across one another.

Increasing the size or number of these microscopic projections by roughening the surfaces generally leads to more friction. If you put sandpaper on the bottom of the file cabinet, it will experience larger frictional forces as it slides across the floor. On the other hand, a microscopically smoother “nonstick” surface, like that used in modern cookware, would let the file cabinet slide easily.

Increasing the number of contact points by squeezing the two surfaces more tightly together also leads to more friction. The microscopic projections simply collide more often. That’s why adding more sheet music to the file cabinet would make it harder to slide. Doubling the file cabinet’s weight would roughly double the number of contact points and make it about twice as hard to move across the floor. A useful rule of thumb is that the frictional forces between two surfaces are proportional to the forces pressing those two surfaces together.

Friction also causes wear when the colliding contact points break one another off. With time, this wear can remove large amounts of material so that even seemingly indestructible stone steps are gradually worn away by foot traffic. The best way to reduce wear between two surfaces (other than to insert a lubricant between them) is to polish them so that they are extremely smooth. The smooth surfaces will still touch at contact points and experience friction as they slide across one another, but their contact points will be broad and round and will rarely break one another off during a collision.

**Check Your Understanding #2: Weight and Friction**

How much harder is it to slide a stack of two identical books across a table than it is to slide just one of those books?

### Static Friction, Sliding Friction, and Traction

There are really two kinds of friction—sliding and static. When two surfaces are moving across one another, **sliding friction** acts to stop them from sliding. But even when those surfaces have the same velocity, **static friction** may act to keep them from starting to slide across one another in the first place.

You find it particularly hard to start the file cabinet sliding across the floor. Contact points between the cabinet and floor have settled into one another, so a small push does nothing. Static friction is always exerting a frictional force that exactly balances your push. Since the net force on the file cabinet is zero, it doesn’t accelerate.

However, the force that static friction can exert is limited. To get the file cabinet moving, you need to give it a mighty shove and thereby exert more horizontal force on it than static friction can exert in the other direction. The net force on the file cabinet is then no longer zero and it accelerates.
Once the file cabinet is moving, static friction is replaced by sliding friction. Because sliding friction acts to bring the file cabinet back to rest, you must push on the cabinet to keep it moving. With the file cabinet sliding across the floor, however, the contact points between the surfaces no longer have time to settle into one another, and they consequently experience weaker horizontal forces. That's why the force of sliding friction is generally weaker than that of static friction and why it's easier to keep the file cabinet moving than it is to get it started.

Both forms of friction are incorporated in the concept of traction—the largest amount of frictional force that the file cabinet can obtain from the floor at any given moment. When the cabinet is stationary, its traction is equal to the maximum amount of force that static friction can exert on it. But once it begins to slide across the floor, its traction reduces to the amount of force that sliding friction exerts.

While the file cabinet's traction is a nuisance that you must overcome, the traction of your shoes on the floor is crucial. Unless you can push against the wall, your shoes are going to need enough traction to provide the horizontal force required to move the file cabinet. Let's hope you're wearing your Doc Martens!

Check Your Understanding #3: Skidding to a Stop

Antilock brakes keep an automobile's wheels from locking and skidding during a sudden stop. Apart from issues of steering, what is the advantage of preventing the wheels from skidding (sliding) on the pavement?

Work, Energy, and Power

There is another difference between static and sliding friction: sliding friction wastes energy. It can't make that energy disappear altogether because energy, as we've seen, is a conserved quantity: it can't be created or destroyed. But energy can be transferred between objects or converted from one form to another. What sliding friction does is convert useful, ordered energy—energy that can easily be used to do work—into relatively useless, disordered energy. This disordered energy is called thermal energy and is the energy we associate with temperature. It's sometimes called internal energy or heat. Sliding friction makes things hotter by turning work into thermal energy.

As we saw in Section 1.3, energy is the capacity to do work and is transferred between objects by doing that work. Energy can also change forms, appearing as either kinetic energy in the motions of objects or as potential energy in the forces between or within those objects. With practice, you can “watch” energy flow through a system just as an accountant watches money flow through a company.

The most obvious form of energy is kinetic energy, the energy of motion. It's easy to see when kinetic energy is transferred into or out of an object. As kinetic energy leaves an object, the object slows down; thus moving water slows down as it turns a gristmill, and a bowling ball slows down as it knocks over bowling pins. Conversely, as kinetic energy enters an object, the object speeds up. A baseball moves faster as you do work on it during a pitch; you’re transferring energy from your body into the baseball, where the energy becomes kinetic energy in the baseball’s motion.

Potential energy is stored in the forces between or within objects, and usually isn’t as visible as kinetic energy. It can take many different forms, some of which appear in
Table 2.2.1. In each case nothing is moving; but because the objects still have a great potential to do work, they contain potential energy.

We measure energy in many common units: joules (J), calories, food Calories (also called kilocalories), and kilowatt-hours, to name only a few. All of these units measure the same thing, and they differ from one another only by numerical conversion factors, some of which can be found in Appendix B. For example, 1 food Calorie is equal to 1000 calories or 4187 J. Thus a jelly donut with about 250 food Calories contains about 1,000,000 J of energy. Since a joule is the same as a newton-meter, 1,000,000 J is the energy you’d use to lift your friend’s file cabinet into the second-floor apartment 200 times (1000 N times 5 m upward is 5000 J of work per trip). No wonder eating donuts is hard on your physique!

Of course, you can eventually use up the energy in a jelly donut; it just takes time. You can only do so much work each second. The measure of how quickly you do work is **power**—the amount of work you do in a certain amount of time, or

\[
\text{power} = \frac{\text{work}}{\text{time}}.
\]

The SI unit of power is the **joule-per-second**, also called the **watt** (abbreviated W). Other units of power include Calories-per-hour and horsepower; like the units for energy, these units differ only by numerical factors, which are again listed in Appendix B. For example, 1 horsepower is equal to 745.7 W. Since a 1-horsepower motor does 745.7 J of work each second, and since it takes 5000 J of work to move the file cabinet to the second floor, that motor has enough power to do the job in about 6.7 s.

Check Your Understanding #4: Apple Overtures

Trace the flow of energy as an archer shoots an apple off the head of her assistant with an arrow.

Friction and Thermal Energy

But what about the **thermal** energy produced by sliding friction? Is thermal energy a new kind of potential energy or an alternative to kinetic energy?

In truth, it’s neither. Thermal energy is actually a mixture of ordinary kinetic and potential energies. But unlike the kinetic energy in a moving ball or the potential energy in an elevated piano, the kinetic and potential energies in thermal energy are disordered at the
Wheels

atomic and molecular level. Thermal energy makes every microscopic particle in an object jiggle independently; at any moment, each particle has its own tiny supply of potential and kinetic energies, and this dispersed energy is collectively referred to as thermal energy.

As you push the file cabinet across the floor, you do work on it, but it doesn’t pick up speed. Instead, sliding friction converts your work into thermal energy, so that the cabinet becomes hotter as the energy you transfer to it disperses among its particles. But while sliding friction easily turns work into thermal energy, there’s no easy way to turn thermal energy back into work. Disorder not only makes things harder to use, but it is also difficult to undo. When you drop your favorite coffee mug on the floor and it shatters into a thousand pieces, the cup is still all there, but it’s disordered and thus much less useful. Just as dropping the pieces on the floor a second time isn’t likely to reassemble your cup, energy converted into thermal energy can’t easily be reassembled into useful, ordered energy.

Sliding friction always converts at least some work into thermal energy. Since two surfaces sliding across one another experience frictional forces that oppose their relative motion, sliding friction does negative work on them; it extracts energy from a sliding object and converts that energy into thermal energy. Thus while you do work on the file cabinet by pushing it across the floor, sliding friction does negative work on it. The file cabinet’s kinetic energy doesn’t change very much, but its thermal energy continues to increase.

In contrast, static friction doesn’t convert work into thermal energy. Since two surfaces experiencing static friction don’t move relative to one another, there is no distance traveled and thus no work done. You can push against the stationary file cabinet all day without doing any work on it. Even if you lift the file cabinet upward with your hands (no easy task), static friction between your hands and the file cabinet’s sides merely assists you in doing work on the file cabinet itself. As you lift the file cabinet upward, all of your work goes into increasing the file cabinet’s gravitational potential energy.

Check Your Understanding #5: Burning Rubber

If you push too hard on your car’s accelerator pedal when the traffic light turns green, your wheels will slip and you’ll leave a black trail of rubber behind. Such a “jackrabbit start” can cause as much wear on your tires as 50 km (31 miles) of normal driving. Why is skidding so much more damaging to the tires than normal driving?

Wheels

You’ve wrestled your friend’s file cabinet out the door of the old apartment and are now dragging it along the sidewalk. You’re doing work against sliding friction the whole way, producing large amounts of thermal energy in both the bottom of the cabinet and the surface of the sidewalk. You’re also damaging both objects, since sliding friction is wearing out their surfaces. The four-drawer file cabinet may be down to three drawers by the time you arrive at the new apartment.

Fortunately, there are mechanical systems that can help you move one object across another without sliding or sliding friction. The classic example is a roller (Fig. 2.2.3). If you place the file cabinet on rollers, those rollers will rotate as the file cabinet moves so that their surfaces never slide across the bottom of the cabinet or the top of the sidewalk. To see how the rollers work, make a fist with one hand and roll it across the palm of your other hand. The skin of one hand doesn’t slide across the skin of the other hand; since this silent motion doesn’t convert work into thermal energy, your skin remains
cool. Now slide your two open palms across one another; this time, sliding friction warms your skin.

Although the rollers don’t experience sliding friction, they do experience static friction. The top of each roller is touching the bottom of the cabinet, and the two surfaces move along together because of static friction; they grip one another tightly until the roller’s rotation pulls them apart. A similar process takes place between the rollers and the top of the sidewalk; static friction exerts torques on the rollers and hence is what makes them rotate in the first place. Again, you can illustrate this behavior with your hands. Try to drag your fist across your open palm. Just before your fist begins to slide, you’ll feel a torque on it. Static friction between the skins of your two hands, acting to prevent sliding, causes your fist to begin rotating just like a roller.

Once you get the file cabinet moving on rollers, you can keep it rolling along the level sidewalk indefinitely. Without any sliding friction, the cabinet doesn’t lose kinetic energy, so it continues at constant velocity without your having to push it. However, the rollers move out from under the file cabinet as it travels, and you frequently have to move a roller from the back of the cabinet to the front. In fact, you need at least three rollers to ensure that the file cabinet never falls to the ground when a roller pops out the back. Although the rollers have eliminated sliding friction, they’ve created another headache—one that makes the prospect of a long trip unappealing. Is there another device that can reduce sliding friction without requiring constant attention?

One alternative would be a four-wheeled cart. The simplest cart rests on fixed poles or axles that pass through central holes or hubs in the four wheels (Fig. 2.2.4). The ground exerts upward support forces on the wheels, the wheels exert upward support forces on the axles, and the axles support the cart and its contents. As the cart moves forward, its wheels turn so that their bottom surfaces don’t slide or skid across the ground; instead, each wheel lowers a portion of its surface onto the sidewalk, leaves it there briefly to experience static friction, and then raises it back off the sidewalk, with a new portion of wheel surface taking its place. Thus there is only static friction between the cart’s wheels and the ground.

Unfortunately, as each wheel rotates, its hub slides across the stationary axle at its center (Fig. 2.2.5). This sliding friction wastes energy and causes wear to both hub and axle. However, the narrow hub moves relatively slowly across the axle so that the work and wear done each second are small. Still, this sliding friction is undesirable and can be reduced significantly by lubricating the hub and axle with “axle grease.”

A better solution is to insert rollers between the hub and axle (Fig. 2.2.6). The result is a roller bearing—a mechanical device that minimizes sliding friction between a hub and an axle. A complete bearing consists of two rings separated by rollers that keep those rings from rubbing against one another. In this case, the bearing’s inner ring is attached to the stationary axle while its outer ring is attached to the spinning wheel hub. The non-driven wheels of an automobile are supported by such bearings on essentially stationary axles. A nondriven bicycle wheel is similarly supported on a stationary axle, but its bearings use balls instead of rollers—ball bearings. When either vehicle starts forward, static friction from the ground exerts torques on its free wheels and they begin to turn.

A car’s driven wheels are also supported by roller bearings, but these bearings act somewhat differently. Because the engine must be able to exert a torque on each driven wheel, those wheels are rigidly connected to their axles. As the engine spins one of these axles, the axle spins its wheel. A bearing prevents the spinning axle from rubbing against the car’s frame. This bearing’s outer ring is attached to the stationary car frame while its inner ring is attached to the spinning axle.

As the driven wheel begins to spin, it experiences static friction with the ground and the ground pushes horizontally on the wheel’s bottom to keep it from skidding. Since that is the only horizontal force on the automobile, the automobile accelerates forward.
Recognizing a good idea when you think of it, you load the file cabinet into the back of your Jaguar XK8 convertible and climb into the driver's seat. The car isn’t quite as responsive as usual because of the added mass, but it’s still able to accelerate respectably. In a few seconds, you’re cruising down the road toward the new apartment and a very grateful friend.

Many antique mechanical watches and clocks proudly proclaim that they have “jewel movements.” Gears in these timepieces turn on axles that are pointed at either end and are supported at those ends by very hard, polished gemstones. What is the advantage of having needlelike ends on an axle and supporting those needles with smooth, hard jewels?

Kinetic Energy

As you near your destination, you begin thinking about the car’s brakes. They’re designed to stop the car by turning its kinetic energy into thermal energy. They’ll perform their task by rubbing stationary brake pads against spinning metal discs, so that sliding friction can transform the energy. Although you’re confident that those brakes are up to the task, just how much kinetic energy are they going to have to convert into thermal energy?

One way is to determine the car’s kinetic energy is to calculate the work its engine did on it while bringing it from rest to its current speed. The result of that calculation is that the moving car’s kinetic energy is equal to one-half of its mass times the square of its speed. This relationship can be written as a word equation:

\[ \text{kinetic energy} = \frac{1}{2} \cdot \text{mass} \cdot \text{speed}^2, \]  

in symbols:

\[ K = \frac{1}{2} \cdot m \cdot v^2, \]  

and in everyday language:

Racing around at twice the speed takes four times the energy.

With you and the file cabinet on board, the XK8 has a mass of about 2000 kg (4400 lbm). At a speed of 100 km/h (62 mph), it has over 770,000 J of kinetic energy. That enormous energy is four times what it would be at 50 km/h (31 mph), so put down your cell phone and drive carefully. The dramatic increase in kinetic energy that results from a modest increase in speed explains why high-speed crashes are far deadlier than those at lower speeds and why that police officer is checking out your car with a radar gun. Red cars get all the attention.

You’re traveling safely within the speed limit and exchange a polite wave with the officer. However, you soon pass another car that has been stopped for a ticket. The light on the nearby police car spins round and round, and rotating objects have kinetic energy, too. Like the kinetic energy of translational motion, the kinetic energy of rotational...
motion depends on the light’s inertia and speed. But for a spinning light, it’s the rotational inertia and rotational speed that matter. The light’s kinetic energy is equal to one-half of its rotational mass times the square of its angular speed. This relationship can be written as a word equation:

\[
\text{kinetic energy} = \frac{1}{2} \cdot \text{rotational mass} \cdot \text{angular speed}^2,
\]

(2.2.2)

in symbols:

\[
K = \frac{1}{2} \cdot I \cdot \omega^2,
\]

and in everyday language:

*It takes a very energetic person to spin his wheels twice as fast.*

With the ticket complete, the police car pulls out into traffic with its light still spinning. The light’s total kinetic energy is now the sum of two parts: translational kinetic energy and rotational kinetic energy. Its translational kinetic energy depends on the speed of the light’s center of mass, which is equal to the police car’s speed through traffic. And its rotational kinetic energy depends on the angular speed at which the light turns about its center of mass.

As the police car disappears in the distance, it occurs to you that the spinning wheels of your car also have rotational kinetic energy that adds to the car’s substantial translational kinetic energy. Still, you trust your brakes. In a few minutes, you arrive at your destination and brake to a stop. Although you’re aware of the added mass as the car decelerates less quickly than usual, the brakes successfully transform the car’s kinetic energy into thermal energy. You’ve reached your goal safely and are now a hero.

**Check Your Understanding #7: Throwing a Fastball**

A typical grade-school pitcher can throw a baseball at 80 km/h (50 mph), but only a few professional athletes have the extraordinary strength needed to throw a baseball at twice that speed. Why is it so much harder to throw the baseball only twice as fast?

**Check Your Figures #1: Blowing in the Wind**

The air in a hurricane travels at 200 km/h (124 mph). How much more kinetic energy does 1 kg of this air have than 1 kg of air moving at only 20 km/h?

**Check Your Figures #2: Playing Around at the Playground**

When children climb onto a playground merry-go-round, they increase its rotational inertia. If the children triple the merry-go-round’s rotational mass, how will they alter the kinetic energy it has when it spins at a certain angular speed?
While car crashes normally aren’t much fun outside of movies or television, there is one delightful exception: bumper cars. For a few minutes, drivers in this amusement park ride race madly about an oval track, deliberately crashing their vehicles into one another and laughing hysterically at the violent impacts. Jolts, jerks, and spins are half the fun, and it’s a wonder that no one gets whiplash. But hidden in the excitement are several important physics concepts that influence everything from tennis to billiards.

Questions to Think About: Why does a stationary car begin rolling forward after being struck by a moving car? What aspects of motion are passed between cars as they collide? Why does your car jolt more when the car that hits it contains two big adults rather than one small child? What would happen if the bumper cars had hard steel bumpers rather than soft rubber ones? Why is your car often set spinning by collisions, and what keeps it spinning?

Experiments to Do: Place a coin on a smooth table and flick a second, identical coin so that it slides along the table and strikes the stationary coin squarely. What happens? Try this experiment again, but now use two coins with different masses. How is the collision different? Does it matter which coin you crash into the other?

Now line up several identical coins so that they touch and slide another coin into one end of this line. How does the collision affect the coin that was originally moving? How does it affect the line of coins? What was transferred among the coins by the collision?

Now stand a coin on its edge and flick it so that it spins rapidly. Did you give it something that keeps it spinning? Why does the coin eventually stop spinning?
Coasting Forward: Linear Momentum

Bumper cars are small, electrically powered vehicles that can turn on a dime and are protected on all sides by rubber bumpers. Each car has only two controls: a pedal that activates its motor and a steering wheel that controls the direction in which the motor pushes the car. Since the car itself is so small, its occupants account for much of the car’s total mass and rotational mass.

Imagine that you have just sat down in one of these cars and put on your safety strap. The other people also climb into their cars, usually one person per car, and the ride begins.

With your car free to move or turn, you quickly become aware of its translational and rotational inertias. The car’s translational inertia makes it hard to start or stop, and its rotational inertia makes it difficult to spin or stop from spinning. While we’ve seen these two types of inertia before, let’s take another look at them and at how they affect your bumper car. This time, we’ll see that they’re associated with two new conserved quantities—linear momentum and angular momentum. It turns out that energy isn’t the only conserved quantity in nature!

When fast-moving bumper cars crash into one another, they exchange more than just energy. Energy is directionless—it’s not a vector quantity—yet these cars seem to be exchanging some quantity of motion that has a direction associated with it. For example, if your car is hit squarely by a rightward moving car, then your car’s motion shifts somewhat rightward in response. Your car is receiving a vector quantity of motion from the other car, a conserved vector quantity known as linear momentum.

Linear momentum, usually just called momentum, is the measure of an object’s translational motion—its tendency to continue moving in a particular direction. Roughly speaking, your car’s momentum indicates which way it’s heading and just how difficult it was to get the car moving with its current velocity.

The car’s momentum is its mass times its velocity and can be written as a word equation:

\[ \text{momentum} = \text{mass} \cdot \text{velocity}, \]  

in symbols:

\[ \mathbf{p} = m \cdot \mathbf{v}, \]

and in everyday language:

It’s hard to stop a fast-moving truck.

Note that momentum is a vector quantity and that it has the same direction as the velocity. As we might expect, the faster your car is moving or the more mass it has, the more momentum it has in the direction of its motion. The SI unit of momentum is the kilogram-meter-per-second (abbreviated kg·m/s).

To physicists, conserved quantities are rare treasures that make it easier to understand otherwise complicated motions. Like all conserved quantities, momentum can’t be created or destroyed. It can only be transferred between objects. Momentum plays a very basic role in bumper cars: the whole point of crashing them into one another is to enjoy the momentum transfers. During each collision, momentum shifts from one car to the other so that they abruptly change their speeds or directions or both. As long as these momentum transfers aren’t too jarring, everyone has a good time.
You’ve stopped your car, so it has zero velocity and zero momentum. To begin moving again, something must transfer momentum to your car. While you could press the pedal and let the motor gradually transfer momentum from the ground to your car, that’s not much fun. Instead, you let two grinning couch potatoes in an overloaded green car slam into you at breakneck speed (Fig. 2.3.1).

The green car was heading westward, and in a few moments your car is moving westward, too, while the green car has slowed significantly. Before you recover from the jolt, your car pounds a child’s car westward and your car slows down abruptly. Finally, its impact with a wall stops the child’s car. Despite disapproving looks from the child’s parents, there’s no harm done. Overall, westward momentum has flowed from the spudmobile to your car, to the child’s car, and into the wall. No momentum has been created or destroyed; you’ve all simply enjoyed passing it along from car to car.

**Check Your Understanding #1: Stuck on the Ice**

Suppose you’re stuck in the middle of a frozen lake, with a surface so slippery that you can’t get any traction. You take off a shoe and throw it toward the southern shore. You find yourself coasting toward the northern shore and soon escape from the lake. Why did this scheme work?

**Check Your Figures #1: Follow That Train!**

The bad guys are getting away in a four-car train and you’re trying to catch them. The train has a mass of 20,000 kg and it’s rolling forward at 22 m/s (80 km/h or 50 mph). What is the train’s momentum?
**Exchanging Momentum in a Collision: Impulses**

Momentum is transferred to a car by giving it an impulse, that is, a force exerted on it for a certain amount of time. When the motor and floor push your bumper car forward for a few seconds, they give your car an impulse and transfer momentum to it. This impulse is the change in your car’s momentum and is equal to the product of the force exerted on the car times the duration of that force. This relationship can be written as a word equation:

\[ \text{impulse} = \text{force} \cdot \text{time}, \]  

(2.3.2)

in symbols:

\[ \Delta p = F \cdot t, \]

and in everyday language:

*The harder and longer you push a bobsled forward at the start of a race, the more momentum it will have when it starts down the hill.*

The more force or the longer that force is exerted, the larger the impulse and the more your car’s momentum changes. Remember that an impulse, like momentum itself, is a vector quantity and has a direction. If your aim is off and the misdirected impulse you obtain from the floor sends you crashing into the wall, don’t say you hadn’t been warned!

Different forces exerted for different amounts of time can transfer the same momentum to a car:

\[ \text{impulse} = \text{large force} \cdot \text{short time} \]
\[ = \text{small force} \cdot \text{long time}. \]  

(2.3.3)

Thus you can get your car moving with a certain forward momentum either by letting the motor and floor push on it with a small forward force of long duration or by letting the colliding green car push on it with a large forward force of short duration.

We can now explain why bumper cars have soft rubber bumpers. If the bumpers were hard steel, the collision between the green car and your car would last only an instant and would involve an enormous forward force. You’d be in need of a neck brace and the services of a personal injury lawyer. However, amusement parks don’t like lawsuits and sensibly limit the car impact forces. To do this, they use rubber bumpers and rather slow-moving cars.

Nonetheless, you can get a pretty good jolt when you collide head-on with another car. Your two cars then start with oppositely directed momenta, and the collision roughly exchanges those momenta between cars. In almost no time, you go from heading forward to heading backward. The impulse that causes this reversal of motion is especially large because it not only stops your forward motion, it also causes you to begin heading backward.

Why should momentum be a conserved quantity? It’s conserved because of Newton’s third law of motion. When one car exerts a force on a second car for a certain amount of time, the second car exerts an equal but oppositely directed force on the first car for exactly the same time. Because of the equal but oppositely directed nature of the two forces, cars that
push on one another receive impulses that are equal in amount but opposite in direction. Since the momentum gained by one car is exactly equal to the momentum lost by the other car, we say that momentum is transferred from one car to the other.

The more mass a car has, the less its velocity changes as a consequence of a momentum transfer. That’s why the green car doesn’t stop completely when it crashes into your car, while your car speeds up dramatically. The green car has so much forward momentum that transferring a fraction of it to your car causes a large change in your car’s velocity. Like a bug being hit by an automobile windshield, your bumper car does most of the accelerating.

![Conserved Quantity: Momentum](image)

**Conserved Quantity: Momentum**  
**Transferred By: Impulse**

**Momentum:** The measure of an object’s translational motion—its tendency to continue moving in a particular direction. Momentum is a vector quantity, meaning that it has a direction. It has no potential form and therefore cannot be hidden; momentum = mass \( \times \) velocity.

**Impulse:** The mechanical means for transferring momentum; impulse = force \( \times \) time.

![Common Misconceptions: Momentum and Force](image)

**Common Misconceptions: Momentum and Force**

**Misconception:** A massive, moving object carries a force with it—the “force of its momentum.”

**Resolution:** Although the impulses that transfer momentum involve forces, momentum itself does not. A moving object carries only momentum, not force. Most importantly, a coasting object is free of any net force. But when that object hits an obstacle, the two will exchange momentum via impulses and those impulses will involve forces.

> **Check Your Understanding #2: Bowling Them Over**

When a beanbag hits the wall, it transfers all of its forward momentum to the wall and comes to a stop. When a rubber ball hits the wall, it transfers all of its forward momentum, comes to a stop, and then rebounds. During the rebound it transfers still more forward momentum to the wall. If you wanted to knock over a weighted bowling pin at the county fair, which would be the more effective projectile: the rubber ball or the beanbag, assuming they have identical masses and you throw them with identical velocities?

> **Check Your Figures #2: Stop That Train!**

The engine of the train you’re chasing (see Check Your Figures #1) has broken down, but it’s still rolling forward. To stop it, you grab onto the last car and begin to drag your boot heels on the ground. The backward force on the train is 200 N. How long will it take you to stop the train?
Spinning in Circles: Angular Momentum

When bumper cars are set spinning during crashes, they are exchanging yet another conserved quantity. Like momentum, it's a conserved vector quantity, but now it's associated with the angular speed and direction of rotational motion around a specific pivot. For example, when your car receives a glancing blow from a car that is circling you clockwise, your car's rotational motion or spin shifts somewhat clockwise in response. Your car is receiving a vector quantity of motion from the other car, a conserved vector quantity known as angular momentum.

Angular momentum is the measure of an object's rotational motion—its tendency to continue spinning about a particular axis. Simply put, your car's angular momentum indicates the direction of its rotation and just how difficult it was to get it spinning with its current angular velocity. The car's angular momentum is its rotational mass times its angular velocity and can be written as a word equation:

\[ \text{angular momentum} = \text{rotational mass} \cdot \text{angular velocity}, \]

in symbols:

\[ L = I \cdot \omega, \]

and in everyday language:

*It's hard to stop a spinning carousel.*

Note that angular momentum is a vector quantity and that it has the same direction as the angular velocity. The faster your car is spinning or the larger its rotational mass, the more angular momentum it has in the direction of its angular velocity. The SI unit of angular momentum is the **kilogram-meter²-per-second** (abbreviated kg·m²/s).

Angular momentum is another conserved quantity, so it can't be created or destroyed; it can only be transferred between objects. For your car to begin spinning, something must transfer angular momentum to it, and your car will then continue to spin until it transfers this angular momentum elsewhere. But to study angular momentum properly, we must pick the pivot about which all the spinning will occur. In the present situation, a good choice for this pivot is your car's initial center of mass.

Your car is stationary again, so it has zero angular velocity and zero angular momentum. Suddenly, a purple car sweeps by and strikes your car a glancing blow (Fig. 2.3.2). Because the purple car was circling your car counterclockwise, it had counterclockwise angular momentum about the pivot. Its impact transfers some of this angular momentum to your car, which begins spinning counterclockwise itself. Since it has given up some of its angular momentum, the purple car circles your car less rapidly. Your car gradually stops spinning as its wheels and friction transfer the angular momentum to the ground and earth. Overall, no angular momentum was created or
destroyed during the collision. Instead, it was transferred from the purple car to your car to the earth.

Check Your Understanding #3: Many Happy Re-Turns

Satellites are often set spinning during launch in order to give them added stability. When astronauts visit these satellites years later, they find them still spinning. Why don’t the satellites stop spinning?

Check Your Figures #3: Want to Go for a Spin?

Spinning satellites are particularly stable. Suppose that the astronauts launching a particular satellite decide to increase its angular velocity by a factor of 5. How will that change affect the satellite’s angular momentum?

Glancing Blows: Angular Impulses

Angular momentum is transferred to a car by giving it an angular impulse, that is, a torque exerted on it for a certain amount of time. When the purple car hits your car and exerts a torque on it briefly, it gives your car an angular impulse and transfers angular momentum to it. This angular impulse is the change in your car’s angular momentum and is equal to the product of the torque exerted on your car times the duration of that torque. This relationship can be written as a word equation:

\[
\text{angular impulse} = \text{torque} \cdot \text{time},
\]

in symbols:

\[
\Delta L = \tau \cdot t,
\]

and in everyday language:

To get a merry-go-round spinning rapidly, you must twist it hard and for a long time.

The more torque or the longer that torque is exerted, the larger the angular impulse and the more your car’s angular momentum changes. Once again, an angular impulse is a vector quantity and has a direction. Had the purple car been circling your car clockwise when it struck the glancing blow, its angular impulse would have been in the opposite direction and you’d be spinning the other way.

Different torques exerted for different amounts of time can transfer the same angular momentum to a car:

\[
\text{angular impulse} = \text{large torque} \cdot \text{short time}
= \text{small torque} \cdot \text{long time}.
\]

(2.3.6)
Thus you can get your car spinning with a certain angular momentum either by letting the motor and floor twist it with a small torque of long duration or by letting the colliding purple car twist it with a large torque of short duration. As with linear momentum, sudden transfers of angular momentum can break things, so the cars are designed to limit their impact torques to reasonable levels. Even so, you may find yourself reaching for the motion sickness bag after a few spinning collisions.

Why should angular momentum be a conserved quantity? Like linear momentum, angular momentum is conserved because of Newton’s third law of motion. In this case, we’re referring to Newton’s third law of rotational motion: if one object exerts a torque on a second object, then the second object will exert an equal but oppositely directed torque on the first object.

Newton’s Third Law of Rotational Motion

For every torque that one object exerts on a second object, there is an equal but oppositely directed torque that the second object exerts on the first object.

When one car exerts a torque on a second car for a certain amount of time, the second car exerts an equal but oppositely directed torque on the first car for exactly the same amount of time. Because of the equal but oppositely directed nature of the two torques, cars that exert torques on one another receive angular impulses that are equal in amount but opposite in direction. Since the angular momentum gained by one car is exactly equal to the angular momentum lost by the other car, we say that angular momentum is transferred from one car to the other.

Because a car’s angular momentum depends on its rotational mass, two different cars may end up rotating at different angular velocities even though they have identical angular momenta. For example, when the purple car hits the overloaded green car and transfers angular momentum to it, the green car’s enormous rotational mass makes it spin relatively slowly. The same sort of behavior occurs with linear momentum, where a car’s mass affects how fast it travels when it’s given a certain amount of linear momentum. But while a bumper car can’t change its mass, it can change its rotational mass. If it does so while it’s spinning, its angular momentum won’t change, but its angular velocity will!

To see this change in angular velocity, consider the overloaded green car. Its two large occupants are disappointed with the ride because their huge mass and rotational mass prevent them from experiencing the intense jolts and spins that you’ve been enjoying. Suddenly they get a wonderful idea. As their car slowly spins, one of them climbs into the other’s lap and the two sit very close to the car’s center of mass. By rearranging the car’s mass this way, they have reduced the car’s overall rotational mass and the car actually begins to spin faster than before.

Since the green car’s mass has been redistributed, it’s no longer a freely turning rigid object covered by Newton’s first law of rotational motion. However, it is freely turning and thus covered by a more general and equally useful rule: an object that is not subject to any outside torques has constant angular momentum. Since the car’s rotational mass has become smaller, its angular velocity must increase in order to keep its angular momentum constant. That’s just what happens. This effect of changing one’s rotational mass explains how an ice skater can achieve an enormous angular velocity by pulling herself into a thin, spinning object on ice (Fig. 2.3.3).
Angular Momentum: The measure of an object's rotational motion—its tendency to continue spinning about a particular axis. Angular momentum is a vector quantity, meaning that it has a direction. It has no potential form and therefore cannot be hidden; angular momentum = rotational mass \times \text{angular velocity}.

Angular Impulse: The mechanical means for transferring angular momentum; angular impulse = torque \times \text{time}.

Summary of Newton's Laws of Rotational Motion

1. A rigid object that is not wobbling and is not subject to any outside torques rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation.

2. The torque exerted on an object that is not wobbling is equal to the product of that object's rotational mass times its angular acceleration. The angular acceleration points in the same direction as the torque.

3. For every torque that one object exerts on a second object, there is an equal but oppositely directed torque that the second object exerts on the first object. 

Note: These laws are the rotational analogs of the translational laws on p. 28.

Check Your Understanding #4: Spinning the Merry-Go-Round

A person who is initially motionless starts a merry-go-round spinning and then returns to being motionless. If angular momentum is truly conserved, what is the source of the angular momentum that the spinning merry-go-round now has?

Check Your Figures #4: Spin Away!

How much longer will it take the astronauts launching the satellite in Check Your Figures #3 to bring it to the faster angular velocity, if they use the initially planned torque?

The Three Conserved Quantities

As you drive your bumper car around the oval, its motion is governed in large part by three conserved quantities: energy, linear momentum, and angular momentum (Table 2.3.1). While you can exchange those quantities with the earth and the power company by steering your car or switching on its motor, most of the interesting exchanges involve collisions.

Each time your car shoves another car forward, your car does work on that other car and transfers energy to it. Each time your car pushes another car northward briefly, your car gives a northward impulse to that other car and transfers northward momentum to it. And each time your car twists another car clockwise about its center of mass, your
You’re backing out of a parking space and accidentally hit a concrete wall. The wall doesn’t move and your car sustains some damage. Did your car transfer any energy or momentum to the wall?

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**Potential Energy and Acceleration**

Shortly before the ride stops, you notice that there is a low point in the floor. After years of use, its metal surface has dented into a bowl-shaped depression and you observe that cars naturally tend to roll into this bowl and accelerate toward its bottom. We’ve seen this tendency to accelerate downhill before with ramps, but now let’s look at it in terms of energy: a car always accelerates in the direction that reduces its total potential energy as quickly as possible. Since a lone car’s only potential energy is gravitational potential energy, it accelerates in such a way as to reduce its gravitational potential energy as quickly as possible: down the steepest route to the bottom of the bowl.

This behavior of accelerating in the direction that reduces total potential energy as quickly as possible is universal. Potential energy and forces are related to one another, so this rule is really just a way to determine the direction of the net force on an object or its parts. An object accelerates in the direction of the net force on it, which is also the direction that will reduce its total potential energy as quickly as possible. This rule is a useful way to determine how motion will proceed: which way...
Bumper Cars

a spring will leap, a chair will tip, or a bumper car will roll. We’ll use it frequently in this book.

Potential Energy and Acceleration
An object accelerates in the direction that reduces its total potential energy as quickly as possible.

Check Your Understanding #6: Heading Down
When you pull a child back on a playground swing and let go, which way does that child accelerate?

Epilogue for Chapter 2

In this chapter we looked at rotating and colliding objects and studied the physical laws that describe their motions. In seesaws, we examined rotational inertia and saw how torques cause angular accelerations. We also noticed how useful it can be to separate an object’s rotational motion from its translational motion. In wheels, we discussed another important type of force, friction, as well as a new type of energy, thermal energy—the energy associated with heat and temperature. In bumper cars, we introduced two more conserved physical quantities: momentum and angular momentum. As we’ll see, following the flows of energy, momentum, and angular momentum between objects often helps in understanding how those objects work.

Explanation: Spinning a Pie Dish

Because the balanced dish has rotational inertia, torques are required both to start it spinning and to stop it from doing so. When you twist the dish with your hand, the torque you exert gives the dish an angular impulse and sets it spinning with a certain amount of angular momentum. If the pivot were truly frictionless and there were no air resistance, the dish would spin indefinitely because it would be unable to get rid of its angular momentum. However, friction in the pivot exerts a small but significant torque that opposes the dish’s motion and gradually slows it down. As this frictional torque transfers angular momentum out of the dish and into the pencil, chair, and earth, the dish turns more and more slowly until it finally comes to a stop. The sharper the pivot and the smaller the contact area between point and dish, the less frictional torque the dish experiences and the longer it spins.

Balancing the dish is easy as long as it’s upside-down. With its edge drooping downward, the dish has relatively little gravitational potential energy and is surprisingly stable. If it begins to tip to one side, the dish’s average height rises and so does its gravitational potential energy. Since objects naturally accelerate in whatever direction lowers their potential energy as quickly as possible, the upside-down dish quickly tips back toward level after being disturbed. In contrast, an upright dish is virtually impossible to balance on a point because any tip will lower its gravitational potential energy and lead quickly to catastrophe. We’ll look at these stabilizing/destabilizing effects more carefully later on in this book.
How Seesaws Work: A seesaw is a rotating toy that works best when it’s almost perfectly balanced, meaning that it experiences zero net torque. The seesaw’s pivot usually passes through its center of mass so that the seesaw balances when it’s not occupied. The riders arrange themselves so that the torques they exert on the seesaw cancel one another completely. The seesaw then experiences zero net torque and zero angular acceleration, and it rotates with constant angular velocity. It either remains motionless or turns steadily in one direction or the other.

To make the seesaw tip back and forth, the riders subtly adjust the torques they exert on the seesaw. They do this either by leaning, thus varying their distances from the pivot, or by pushing against the ground with their feet, thus varying the forces they exert on the seesaw. In either case, they unbalance the seesaw, and it experiences both a net torque and an angular acceleration. By rhythmically changing the net torque on the seesaw, the riders cause it to rotate back and forth.

How Wheels Work: Wheels facilitate motion by eliminating or reducing sliding friction between an object and a surface. The wheels convey the support forces needed to hold the object up but allow the object to move without sliding. As a cart with freely turning wheels moves along the surface, static friction between each wheel and the surface exerts a torque on that wheel and causes it to turn. However, rubbing may occur between the wheel’s hub and the axle, where sliding friction can waste energy and cause wear. To eliminate this sliding friction, roller or ball bearings are often used.

The torque that causes a powered wheel on a vehicle to turn comes from an engine by way of an axle. In this case, static friction between the outside of the wheel and the ground exerts a torque on the wheel that opposes the torque from the engine. This static frictional force also contributes to the net force on the vehicle and causes it to accelerate.

Once supported on wheels and bearings, objects can move freely and can retain linear momentum, angular momentum, and energy for long periods of time. By eliminating sliding friction, wheels can also keep objects from converting ordered energy into thermal energy. Wheels allow vehicles to hold onto these conserved quantities for extended periods and make transportation far more practical.

How Bumper Cars Work: Since they start from rest, bumper cars must obtain their initial momenta and angular momenta from the ground and their initial kinetic energies from the power company. They do this with the help of motors and wheels, which gradually transfer energy, momentum, and angular momentum into the cars.

Once the cars are moving, they can begin to exchange those conserved quantities by way of collisions. Each impact usually changes the cars’ speeds and directions of travel in a manner that may seem rather complicated. However, following the exchanges of momentum, angular momentum, and energy often makes it easier to understand these collisions.

Cars containing massive riders respond weakly when collisions transfer momentum and angular momentum to them. That’s because their large masses and rotational masses minimize their changes in velocity and angular velocity. Because of their small masses, children experience the wildest rides.
Important Laws and Equations

1. Newton’s First Law of Rotational Motion: A rigid object that is not wobbling and is not subject to any outside torques rotates at a constant angular velocity, turning equal amounts in equal times about a fixed axis of rotation.

\[ \text{torque} = \text{rotational mass} \cdot \text{angular acceleration}. \] (2.1.2)

The angular acceleration points in the same direction as the torque. This law doesn’t apply to objects that are wobbling.

2. Newton’s Second Law of Rotational Motion: The torque exerted on an object is equal to the product of that object’s rotational mass times its angular acceleration, or

\[ \text{impulse} = \text{force} \cdot \text{time}. \] (2.3.2)

3. Relationship between Force and Torque: The torque produced by a force is equal to the product of the lever arm times the component of that force perpendicular to the lever arm, or

\[ \text{kinetic energy} = \frac{1}{2} \cdot \text{mass} \cdot \text{speed}^2. \] (2.2.1)

4. Kinetic Energy: An object’s translational kinetic energy is one-half of its mass times the square of its speed, or

An object’s rotational kinetic energy is one-half of its rotational mass times the square of its angular speed, or

\[ \text{kinetic energy} = \frac{1}{2} \cdot \text{rotational mass} \cdot \text{angular speed}^2. \] (2.2.2)

5. Linear Momentum: An object’s linear momentum is its mass times its velocity, or

\[ \text{linear momentum} = \text{mass} \cdot \text{velocity}. \] (2.3.1)

6. The Definition of Impulse: The impulse given to an object is equal to the product of the force exerted on that object times the length of time that force is exerted, or

\[ \text{angular momentum} = \text{rotational mass} \cdot \text{angular velocity}. \] (2.3.4)

8. The Definition of Angular Impulse: The angular impulse given to an object is equal to the product of the torque exerted on that object times the length of time that torque is exerted, or

\[ \text{angular impulse} = \text{torque} \cdot \text{time}. \] (2.3.5)

9. Newton’s Third Law of Rotational Motion: For every torque that one object exerts on a second object, there is an equal but oppositely directed torque that the second object exerts on the first object.

10. Potential Energy and Acceleration: An object accelerates in the direction that reduces its total potential energy as quickly as possible.

Check Your Understanding—Answers

Section 2.1 SEESAWS

1. The lazy Susan undergoes rotational motion while the dessert cart undergoes translational motion.

\textbf{Why:} The lazy Susan has a fixed pivot at its center. This pivot never goes anywhere, no matter how you rotate the lazy Susan. In contrast, the dessert cart moves about the room and has no fixed point. The server can rotate the dessert cart when necessary, but its principal motion is translational.

2. It will continue to spin at a steady pace about a fixed rotational axis (although friction with the water will gradually slow the ball’s rotation).

\textbf{Why:} Because the basketball is free of torques, the outside influences that affect rotational motion, it has a constant angular velocity. If you spin the basketball, it will continue to spin about whatever axis you chose. If you don’t spin the basketball, its angular velocity will be zero and it will remain stationary.

3. Yes. His center of mass falls smoothly, obeying the rules governing falling objects. As he falls, his body rotates at constant angular velocity about his center of mass.

\textbf{Why:} Like a thrown football or tossed baton, the diver is a rigid, rotating object. His motion can be separated into translational motion of his center of mass (it falls) and rotational motion about his center of mass (he rotates about it at constant angular velocity). While the diver may never think of his motion in these terms, he is aware intuitively of the need to handle both his rotational and translational motions carefully. Hitting the water with his chest because he mishandled his rotation isn’t much more fun than hitting the board because he mishandled his translation.
4. The full merry-go-round has a huge rotational mass. 
   **Why:** Starting or stopping a merry-go-round involves angular acceleration. As the pusher, you exert a torque on the merry-go-round and it undergoes angular acceleration. But this angular acceleration depends on the merry-go-round’s rotational mass, which in turn depends on how much mass it has and how far that mass is from the axis of rotation. With many children adding to the merry-go-round’s rotational mass, its angular acceleration tends to be small.

5. The closer the cardboard is to the pivot, the more force it must exert on the scissors to produce enough torque to keep the scissors from rotating closed. When the cardboard is unable to produce enough torque, the scissors cut through it.
   **Why:** When you place paper close to the pivot of a pair of scissors, you are requiring that paper to exert enormous forces on the scissors to keep them from rotating closed. Rotations are started and stopped by torques, and forces exerted close to the pivot exert relatively small torques.

6. About 1000 N (220 lbf).
   **Why:** Since the nail is 10 times closer to the pivot, the nail must exert 10 times the force on the hammer to create the same magnitude of torque as you do pulling on the handle. As the nail pulls on the hammer, the hammer pulls on the nail. Although the wood exerts frictional forces on the nail to keep it from moving, the extracting force overwhelms this friction and the nail slides slowly out of the wood.

7. A substantial shift in the cargo’s position during a storm can unbalance the ship, creating a net torque on the ship, and cause it to begin rotating about the effective pivot. The ship may then capsize.
   **Why:** Although most boats can compensate for some amount of cargo imbalance, shifting cargo can easily flip even a fairly stable boat. It happens frequently in real life, often with fatal consequences. Some boats, particularly canoes and racing shells, are notoriously sensitive to unbalanced loading and are easily flipped by careless or moving occupants.

**Section 2.2 WHEELS**

1. Friction is pushing the glass uphill.
   **Why:** The glass is sliding downhill across the top of the stationary table. Since friction always opposes relative motion, it pushes the glass uphill, in the direction opposite its motion.

2. About twice as hard.
   **Why:** The frictional forces between the table and books are roughly proportional to the forces pressing them together. The book’s weight is what pushes them together. When you effectively double the book’s weight, by stacking a second book on top of it, you double the frictional forces between the table and the books.

3. If the wheels continue to turn, they experience static friction. If they lock and begin to skid, they experience sliding friction. Since the traction provided by static friction is greater than that provided by sliding friction, the car will decelerate faster if the wheels don’t skid.
   **Why:** For a rapid stop, the car needs the maximum possible force in the direction opposite its velocity. The most effective way to obtain that stopping force from the road is with static friction between the turning wheels and the pavement. Sliding friction, the result of skidding tires, is much less effective at stopping the car, wears out the tires, and diminishes the driver’s ability to steer the vehicle.

4. The archer does work on the string and bow as she draws the arrow back. (Chemical energy from her body is transferred to the bow, where it is stored as elastic potential energy.) As she releases the arrow, the string and bow do work on the arrow. (Elastic potential energy is transferred to the arrow, where it becomes kinetic energy.) Finally, the arrow does work on the apple, knocking it off the head of the assistant. (Kinetic energy is transferred from the arrow to the apple.)
   **Why:** Because energy is conserved, we could in principle follow it back to the origins of the universe. Whatever energy we see around us now was somewhere in our universe yesterday, last week, and a million years ago, although its form may have changed. It will still be in our universe next year, too, but maybe not in so useful a form.

5. Normal driving involves mostly static friction because the surfaces of the tires don’t slide across the pavement. Skidding involves sliding friction as the tire surfaces move independently of the pavement. Because it involves sliding friction, skidding creates thermal energy and damages the tires.
   **Why:** The expression “burn rubber” is an appropriate name for skidding during a jackrabbit start. Substantial thermal energy is produced, and a trail of hot rubber is left on the pavement behind the car. At drag races, the frictional heating that results from skidding at the start can be so severe that the tires actually catch on fire.

6. Because all the supporting forces are very close to the axis of rotation, the jewels exert almost zero torque on the axle. The axle turns remarkably freely.
   **Why:** Mechanical timepieces need almost ideal motion to keep accurate time. One of the best ways to allow a rotating object free movement is to support it exactly on the axis of rotation, where the support can’t exert torque on the object.

7. Doubling the speed of the baseball requires quadrupling the energy transferred to it by the pitcher.
   **Why:** To throw a 160-km/h (100-mph) fastball, a major league pitcher must put four times as much kinetic energy into both the ball and his arm as when pitching an 80-km/h slow ball. He also pitches the fastball in half the time needed to pitch the slow ball. Overall, he must do four times as much work throwing a fastball and he must do that work in one-half the time. That means the pitcher produces eight times as
Bumper Cars

much power while throwing a fastball as when throwing a slow ball. No wonder amateurs have trouble duplicating that feat.

Section 2.3 BUMPER CARS

1. By transferring southward momentum to the shoe, you would obtain northward momentum.
Why: Initially, both you and your shoe have zero momentum. But when you throw the shoe southward, you give it southward momentum. Since the only source of that southward momentum is you, you must have lost southward momentum. A negative amount of southward momentum is actually northward momentum, and thus you coast northward. Interestingly enough, the total momentum of you and the shoe hasn’t changed. It’s still zero, as it must be because momentum is conserved. It has simply been redistributed.

2. The bouncy rubber ball would be more effective.
Why: Either projectile will transfer all of its original momentum to the bowling pin while coming to a stop. But then the bouncy rubber ball will bounce back and continue to exert a force on the bowling pin. The impulse (force \cdot time) delivered by the rubber ball will be greater than that delivered by the beanbag because the ball will exert its forward force for a longer time (during stopping and rebounding). The ball will rebound with its momentum reversed, having transferred roughly twice its original momentum to the pin.

3. The satellites are unable to get rid of their angular momentum.
Why: Because of their extreme isolation, orbiting satellites have nothing with which to exchange angular momentum. The angular momentum given to them at launch stays with them indefinitely, so they continue to spin for decades.

4. It came from the entire earth.
Why: Because the person stood on the earth as he started the merry-go-round spinning, he transferred angular momentum from the earth to the merry-go-round. The merry-go-round spins in one direction, and the earth’s rotation changes ever so slightly in the other direction. Because the earth is so huge and has such an enormous rotational mass, its slight change in rotation is undetectable.

5. Your car transferred momentum but no energy.
Why: To transfer momentum to the wall, your car must give it an impulse: it must push on the wall for an amount of time. It did so and thus transferred all its backward momentum to the wall. But to transfer energy to the wall, your car must do work on it: it must push on the wall as the wall moves in the direction of that push. But the wall is immobile, so your car couldn’t do work on it. Instead, the car’s energy stayed in the car, where it caused damage.

6. The child accelerates forward, in the direction that will reduce the child’s potential energy as quickly as possible.
Why: The child has only one form of potential energy—gravitational potential energy. This gravitational potential energy is lowest when the child is directly below the swing’s supporting bar. The child accelerates forward because that will put the child below the support as quickly as possible.

Check Your Figures—Answers

Section 2.1 SEESAWS

1. About 10 times as much torque.
Why: To keep the angular acceleration in Eq. 2.1.1 unchanged while increasing the rotational mass by a factor of 10, the torque must also increase by a factor of 10. Solid tires are extremely difficult to spin or to stop from spinning, which is why automobiles use hollow tires.

2. Five times as much torque as before.
Why: The pipe increases the wrench’s lever arm by a factor of 5, from 0.2 meter to 1.0 meter. According to Eq. 2.1.3, the same force exerted five times as far from the pivot will produce five times as much torque about that pivot. Extending the handle of a lever-like tool is a common technique to increase the available torque, although it can be hazardous for both the tool and its user. Some tools that are designed for such extreme use come with removable handle extensions.

Section 2.2 WHEELS

1. One hundred times as much kinetic energy.
Why: Because kinetic energy is proportional to speed squared, the kilogram of air in the hurricane moves 10 times as fast but has 100 times as much kinetic energy as the slower moving air. This enormous increase in energy is what makes a hurricane’s wind dangerous. The air’s terrific speed also brings large quantities of it to you quickly, so that the wind power arriving each second is overwhelming.

2. The children will triple the merry-go-round’s kinetic energy.
Why: The kinetic energy of a spinning object is proportional to its rotational mass. Since the children triple the merry-go-round’s rotational mass, they triple its kinetic energy.
Section 2.3 BUMPER CARS

1. **440,000 kg·m/s, in the forward direction.**
   **Why:** You can use Eq. 2.3.1 to calculate the train’s momentum from its mass and velocity:
   \[
   \text{linear momentum} = 20,000 \text{ kg} \cdot 22 \text{ m/s} = 440,000 \text{ kg} \cdot \text{m/s}.
   \]
   That momentum is in the same direction the train is moving, the forward direction.

2. **2200 s, so kiss your boot heels goodbye!**
   **Why:** To stop the train, you must give it a backward impulse that completely cancels its forward momentum. Since its forward momentum is 440,000 kg·m/s, the backward impulse must be 440,000 kg·m/s. Since 200 N can also be written as 200 kg·m/s², we can use Eq. 2.3.2 to find the time:
   \[
   \text{time} = \frac{440,000 \text{ kg} \cdot \text{m/s}}{200 \text{ kg} \cdot \text{m/s}^2} = 2200 \text{ s}.
   \]

3. **The angular momentum will increase by a factor of 5.**
   **Why:** Because the satellite’s angular momentum is proportional to its angular velocity, spinning it five times faster will increase its angular momentum by that same factor.

4. **It will take them five times as long.**
   **Why:** To reach the new, faster angular velocity, the astronauts will need an angular impulse that’s five times as large as originally planned. Since they will be using the same torque, they will have to exert that torque for five times as long.

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**Exercises**

1. The chairs in an auditorium aren’t all facing the same direction. How could you describe their angular positions in terms of a reference orientation and a rotation?

2. When an airplane starts its propellers, they spin slowly at first and gradually pick up speed. Why does it take so long for them to reach their full rotational speed?

3. A mechanic balances the wheels of your car to make sure that their centers of mass are located exactly at their geometrical centers. Neglecting friction and air resistance, how would an improperly balanced wheel behave if it were rotating all by itself?

4. An object’s center of mass isn’t always inside the object, as you can see by spinning it. Where is the center of mass of a boomerang or a horseshoe?

5. Why is it hard to start the wheel of a roulette table spinning, and what keeps it spinning once it’s started?

6. Why can’t you open a door by pushing its doorknob directly toward or away from its hinges?

7. Why can’t you open a door by pushing on its hinged side?

8. It’s much easier to carry a weight in your hand when your arm is at your side than it is when your arm is pointing straight out in front of you. Use the concept of torque to explain this effect.

9. A gristmill is powered by falling water, which pours into buckets on the outer edge of a giant wheel. The weight of the water turns the wheel. Why is it important that those buckets be on the wheel’s outer edge?

10. How does the string of a yo-yo get the yo-yo spinning?

11. One way to crack open a walnut is to put it in the hinged side of a door and then begin to close the door. Why does a small force on the door produce a large force on the shell?

12. A common pair of pliers has a place for cutting wires, bolts, or nails. Why is it so important that this cutter be located very near the pliers’ pivot?

13. You can do push-ups with either your toes or your knees acting as the pivot about which your body rotates. When you pivot about your knees, your feet actually help you to lift your head and chest. Explain.

14. Tightrope walkers often use long poles for balance. Although the poles don’t weigh much, they can exert substantial torques on the walkers to keep them from tipping and falling off the ropes. Why are the poles so long?

15. Some racing cars are designed so that their massive engines are near their geometrical centers. Why does this design make it easier for these cars to turn quickly?
16. How does a bottle opener use mechanical advantage to pry the top off a soda bottle?

17. A jar-opening tool grabs onto a jar’s lid and then provides a long handle for you to turn. Why does this handle’s length help you to open the jar?

18. When you climb out on a thin tree limb, there’s a chance that the limb will break off near the trunk. Why is this disaster most likely to occur when you’re as far out on the limb as possible?

19. How does a crowbar make it easier to lift the edge of a heavy box a few centimeters off the ground?

20. The basket of a wheelbarrow is located in between its wheel and its handles. How does this arrangement make it relatively easy for you to lift a heavy load in the basket?

21. Skiers often stop by turning their skis sideways and skidding them across the snow. How does this trick remove energy from a skier, and what happens to that energy?

22. A horse does work on a cart it’s pulling along a straight, level road at a constant speed. The horse is transferring energy to the cart, so why doesn’t the cart go faster and faster? Where is the energy going?

23. Explain why a rolling pin flattens a piecrust without encountering very much sliding friction as it moves.

24. Professional sprinters wear spikes on their shoes to prevent them from sliding on the track at the start of a race. Why is energy wasted whenever a sprinter’s foot slides backward along the track?

25. A yo-yo is a spool-shaped toy that spins on a string. In a sophisticated yo-yo, the end of the string forms a loop around the yo-yo’s central rod so that the yo-yo can spin almost freely at the end of the string. Why does the yo-yo spin longest if the central rod is very thin and very slippery?

26. As you begin pedaling your bicycle and it accelerates forward, what is exerting the forward force that the bicycle needs to accelerate?

27. When you begin to walk forward, what exerts the force that allows you to accelerate?

28. If you are pulling a sled along a level field at constant velocity, how does the force you are exerting on the sled compare to the force of sliding friction on its runners?

29. Why does putting sand in the trunk of a car help to keep the rear wheels from skidding on an icy road?

30. When you’re driving on a level road and there’s ice on the pavement, you hardly notice that ice while you’re heading straight at a constant speed. Why is it that you only notice how slippery the road is when you try to turn left or right, or to speed up or slow down?

31. Describe the process of writing with chalk on a blackboard in terms of friction and wear.

32. Falling into a leaf pile is much more comfortable than falling onto the bare ground. In both cases you come to a complete stop, so why does the leaf pile feel so much better?

33. In countless movie and television scenes, the hero punches a brawny villain who doesn’t even flinch at the impact. Why is the immovable villain a Hollywood fantasy?

34. Why can’t an acrobat stop himself from spinning while he is in midair?

35. While a gymnast is in the air during a jump, which of the following quantities must remain constant for her: velocity, momentum, angular velocity, or angular momentum?

36. If you sit in a good swivel chair with your feet off the floor, the chair will turn slightly as you move about but will immediately stop moving when you do. Why can’t you make the chair spin without touching something?

37. When a star runs out of nuclear fuel, gravity may crush it into a neutron star about 20 km (12 miles) in diameter. While the star may have taken a year or so to rotate once before its collapse, the neutron star rotates several times a second. Explain this dramatic increase in angular velocity.

38. A toy top spins for a very long time on its sharp point. Why does it take so long for friction to slow the top’s rotation?

39. It’s easier to injure your knees and legs while hiking downhill than while hiking uphill. Use the concept of energy to explain this observation.

40. When you first let go of a bowling ball, it’s not rotating. But as it slides down the alley, it begins to rotate. Use the concept of energy to explain why the ball’s forward speed decreases as it begins to spin.

41. Firefighters slide down a pole to get to their trucks quickly. What happens to their gravitational potential energy, and how does it depend on the slipperiness of the pole?
Problems

1. When you ride a bicycle, your foot pushes down on a pedal that’s 17.5 cm (0.175 m) from the axis of rotation. Your force produces a torque on the crank attached to the pedal. Suppose that you weigh 700 N. If you put all your weight on the pedal while it’s directly in front of the crank’s axis of rotation, what torque do you exert on the crank?

2. An antique carousel that’s powered by a large electric motor undergoes constant angular acceleration from rest to full rotational speed in 5 seconds. When the ride ends, a brake causes it to decelerate steadily from full rotational speed to rest in 10 seconds. Compare the torque that starts the carousel to the torque that stops it.

3. When you start your computer, the hard disk begins to spin. It takes 6 seconds of constant angular acceleration to reach full speed, at which time the computer can begin to access it. If you wanted the disk drive to reach full speed in only 2 seconds, how much more torque would the disk drive’s motor have to exert on it during the starting process?

4. An electric saw uses a circular spinning blade to slice through wood. When you start the saw, the motor needs 2 seconds of constant angular acceleration to bring the blade to its full angular velocity. If you change the blade so that the rotating portion of the saw now has three times its original rotational mass, how long will the motor need to bring the blade to its full angular velocity?

5. When the saw in Problem 4 slices wood, the wood exerts a 100-N force on the blade, 0.125 m from the blade’s axis of rotation. If that force is at right angles to the lever arm, how much torque does the wood exert on the blade? Does this torque make the blade turn faster or slower?

6. When you push down on the handle of a doll-like wooden nutcracker, its jaw pivots upward and cracks a nut. If the point at which you push down on the handle is five times as far from the pivot as the point at which the jaw pushes on the nut, how much force will the jaw exert on the nut if you exert a force of 20 N on the handle? (Assume all forces are at right angles to the lever arms involved.)

7. Some special vehicles have spinning disks (flywheels) to store energy while they roll downhill. They use that stored energy to lift themselves uphill later on. Their flywheels have relatively small rotational masses but spin at enormous angular speeds. How would a flywheel’s kinetic energy change if its rotational mass were five times larger but its angular speed were five times smaller?

8. What is the momentum of a fly if it’s traveling 1 m/s and has a mass of 0.0001 kg?

9. Your car is broken, so you’re pushing it. If your car has a mass of 800 kg, how much momentum does it have when it’s moving forward at 3 m/s (11 km/h)?

10. You begin pushing the car forward from rest (see Problem 9). Neglecting friction, how long will it take you to push your car up to a speed of 3 m/s on a level surface if you exert a constant force of 200 N on it?

11. When your car is moving at 3 m/s (see Problems 9 and 10), how much translational kinetic energy does it have?

12. No one is driving your car (see Problems 9, 10, and 11) and it crashes into a parked car at 3 m/s. Your car comes to a stop in just 0.1 s. What force did the parked car exert on it to stop it that quickly?

13. You’re at the roller-skating rink with a friend who weighs twice as much as you do. The two of you stand motionless in the middle of the rink so that your combined momentum is zero. You then push on one another and begin to roll apart. If your momentum is then 450 kg·m/s to the left, what is your friend’s momentum?